■ ǖǀƢǂƾǀƣƪƨƫƢ : (ƾƧƢƼǀƾ)

(1 $(1): r^2-6r+9=0$ ǂƳ $y''-6y'+9y=0$: ƽƾǀƾƢƪƾƢ ƞƾǀƾƢƪƾƢ

$\Delta=(-6)^2-4\times 9=0$ ƛƳ ǀƳǂƾƾǀ ƛ .

: ǂƳ ƽƾǀƾƢƪƾƢ ƞƾǀƾƢƪƾƢ ǀǂƾǀƛ ǀ $r_0=\dfrac{6}{2}=3$: ƛƳ ƽƾǀƛ ƾǀ ǀƳǀ ƾǀƢ

ǀ 0,75 . $y: x\mapsto (Ax+B)e^{3x}\ /(A,B)\in\mathbb{R}^2$

(2 ƣ- ƞƣƾƾ : $u: x\mapsto x^2e^{3x}$ ƽƾǀƽ ƾǀƛǀƾƢ ƞǀƾ ƾǀ $\mathbb{R}$ ƛ ƞǀƣƾ :

$$\forall x\in\mathbb{R}:\begin{cases}u'(x)=(3x^2+2x)e^{3x}\\ u''(x)=(9x^2+12x+2)e^{3x}\end{cases}$$

$$u''(x)-6u'(x)+9u(x)=\left[9x^2+12x+2-6(3x^2+2x)+9x^2\right]e^{3x}\ :\text{ƾǀƢ}$$

$$u''(x)-6u'(x)+9u(x)=2e^{3x}\ :\text{ƾǀ ǀǀ}$$

ǀ 0,75 : ƽƾǀƾƢƪƾƢ ƞƾǀƾƢƪƾƢ ƾǀǀ ǀƾ $u: x\mapsto x^2e^{3x}$ ƞƢƣƢǀ ǂƢǀƢǀ ƛ

. $(E): y''-6y'+9y=2e^{3x}$

ƞ- ƾǀƳ ƞǀƳƪƾƢ (E) ƛƳ : $z: x\mapsto (x^2+Ax+B)e^{3x}\ /(A,B)\in\mathbb{R}^2$

ǀ 0,5

■ ƞǀƾƢƾƫƢ ƾƣƪƨƫƢ : (ƾƧƢ ƾƧƢ)

(1 ƞǀƾƾƢƪƾ ƞǀƾƳƪƾƢ $z^2-2\sqrt{3}(1+i)z+8i=0$ ƛƳ: $\Delta'=3(1+i)^2-8i=-2i$

: ǀƾƳ ƽƾǀƾƢƪƾƢ ƞƾǀƾƢƪƾƢ ǂƢǀƾ ǂƢǀƾƢǀƛ ǀ $\Delta'=(1-i)^2$: ƾǀƢ

ǀ 0,75 . $z_2=(\sqrt{3}-1)+i(\sqrt{3}+1)$ ƛ $z_1=(\sqrt{3}+1)+i(\sqrt{3}-1)$

(2 ƣ- ƞƣƾƾ :

$$z_1^2=\left[(\sqrt{3}+1)+i(\sqrt{3}-1)\right]^2$$

$$=(\sqrt{3}+1)^2-(\sqrt{3}-1)^2+2i\left(\sqrt{3}^2-1^2\right)=4\sqrt{3}+4i=4(\sqrt{3}+i)$$

$$z_1^2=4(\sqrt{3}+i)\ :\text{ƾǀƢ}$$

$$z_2=(\sqrt{3}-1)+i(\sqrt{3}+1)=i\left[(\sqrt{3}+1)-i(\sqrt{3}-1)\right]\ \text{ƛ}$$

(ǀ 1) . $z_2=i\overline{z_1}$: ǀƾ ǀǀ

ƣ- ƞƣƾƾ : $\sqrt{3}+i=2\left(\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i\right)=2\left(\cos\left(\dfrac{\pi}{6}\right)+i\sin\left(\dfrac{\pi}{6}\right)\right)$

(ǀ 0,25) . $4(\sqrt{3}+i)=\left[8,\dfrac{\pi}{6}\right]$: ƾǀƢ

ƞ- ƞƣƾƾ : $z_1=\left[\sqrt{8},\dfrac{\pi}{12}\right]=\left[2\sqrt{2},\dfrac{\pi}{12}\right]$: ƾǀƢ ǀ $z_1^2=4(\sqrt{3}+i)=\left[8,\dfrac{\pi}{6}\right]$

$$z_2=i\overline{z_1}=\left[1,\frac{\pi}{2}\right]\times\left[2\sqrt{2},-\frac{\pi}{12}\right]=\left[2\sqrt{2},\frac{\pi}{2}-\frac{\pi}{12}\right]\ \text{ƛ}$$

(ǀ 1) . $z_2=\left[2\sqrt{2},\dfrac{5\pi}{12}\right]$: ƾǀƢ

(3 ƞƣƾƾ : $\arg\left(\dfrac{z_2}{z_1}\right)\equiv\arg(z_2)-\arg(z_1)[2\pi]$

. $\arg\left(\dfrac{z_2}{z_1}\right)\equiv\dfrac{\pi}{3}[2\pi]$: ǀƾ ǀǀ $\arg\left(\dfrac{z_2}{z_1}\right)\equiv\dfrac{5\pi}{12}-\dfrac{\pi}{12}[2\pi]$: ƾǀƢ

$\overline{(\overrightarrow{OA},\overrightarrow{OB})}\equiv\dfrac{\pi}{3}[2\pi]$: ǀƾƣ $\overline{(\overrightarrow{OA},\overrightarrow{OB})}\equiv\arg\left(\dfrac{z_2}{z_1}\right)[2\pi]$: ǀƾ ǀƾǀ ƛ

$OB=OA=2\sqrt{2}$: ƾǀƢ ǀ $|z_2|=|z_1|=2\sqrt{2}$: ƞƣƾƾ ƛ

(ǀ 1) . ƽƾ ƾƾƢ ǀƛǀƾǀ OAB ƾǀƾƢ ǀƾ ǂƾǀƾ ƾƾǀ

■ [illegible] ([illegible])

1) [illegible] (OA) [illegible] $(O,\overrightarrow{OA})$: $\begin{cases} x = 0+1t \\ y = 0-1t \ / t\in\mathbb{R} \\ z = 0+3t \end{cases}$

[illegible] : $(OA):\begin{cases} x = t \\ y = -t \ / t\in\mathbb{R} \\ z = 3t \end{cases}$. ([illegible] 0,5)

b – [illegible] $(Q)\perp(OA)$ [illegible] (Q) [illegible] $\overrightarrow{OA}$ [illegible] :

$$M(x,y,z)\in(Q)\Leftrightarrow \overrightarrow{AM}.\overrightarrow{OA}=0$$
$$\Leftrightarrow (x-1)-(y+1)+3(z-3)=0$$
$$\Leftrightarrow x-y+3z-11=0$$

[illegible] : $(Q): x-y+3z-11=0$ ([illegible] 0,75)

c) [illegible] $\vec{n}(1,-1,3)$ [illegible] (P) [illegible] (Q) : $(P)//(Q)$.

([illegible] 0,25)

2) – a [illegible] (Q) [illegible] (S) [illegible] A [illegible] : $(\Omega A)\perp(Q)$

[illegible] : $(OA)\perp(Q)$ [illegible] $(OA)//(\Omega A)$ [illegible] : $(OA)=(\Omega A)$

[illegible] : $\Omega\in(OA)$.

$$\Omega(a,b,c)\in(OA)\Leftrightarrow \exists t\in\mathbb{R}/\begin{cases} a=t \\ b=-t \\ c=3t \end{cases}$$ [illegible]

[illegible] : $b=-a$ [illegible] $c=3a$. ([illegible] 0,75)

b – [illegible] (Q) [illegible] (S) ([illegible] Ω [illegible] $r=\Omega A$)

[illegible] O [illegible] $\sqrt{33}$ [illegible] : $OA^2+\Omega O^2=\Omega A^2$

[illegible] : $33+\Omega O^2=\Omega A^2$ ([illegible] $OA=\sqrt{33}$ [illegible] $A\in T$)

[illegible] : $\Omega A^2-\Omega O^2=33$.

[illegible] : $\Omega A^2-\Omega O^2=(a-1)^2+(b+1)^2+(c-3)^2-(a^2+b^2+c^2)$

[illegible] : $\Omega A^2-\Omega O^2=-2a+1+2b+1-6c+9$

[illegible] : $\Omega A^2-\Omega O^2=33\Leftrightarrow -2a+1+2b+1-6c+9=33$

[illegible] : $2a-2b+6c+22=0$

[illegible] : $a-b+3c=-11$ ([illegible] 1,25)

c – [illegible] Ω [illegible] :

$\begin{cases} a-b+3c=-11 \\ b=-a \\ c=3a \end{cases}$ [illegible] : $\begin{cases} 11a=-11 \\ b=-a \\ c=3a \end{cases}$

[illegible] : $\Omega(-1,1,-3)$. ([illegible] 0,5)

■ [illegible] ([illegible] **10**)

I- [illegible] g [illegible] $[0,+\infty[$ [illegible] : $g(x)=\ln(1+x)-x$.

1) [illegible] : $\forall x\in[0,+\infty[: g'(x)=(-x)'+(\ln(1+x))'$

$$=-1+\frac{(1+x)'}{1+x}$$
$$=-1+\frac{1}{1+x}$$
$$=\frac{-x}{x+1}$$

[illegible] : $g'(x)\prec 0$ [illegible] x [illegible] $]0,+\infty[$ [illegible] g [illegible]

$[0,+\infty[$. ([illegible] 0,75)

f – [illegible] g [illegible] $[0,+\infty[$ [illegible] :

$g(0)=\ln 1=0$ [illegible] $\forall x\in[0,+\infty[: g(x)\le g(0)$

[illegible] : $\forall x\in[0,+\infty[: g(x)\le 0$.

([illegible] 0,25)

(2 g [illegible] $[0,+\infty[$: $\forall x \in]0,+\infty[: g(x) \prec g(0)$

$\forall x \in]0,+\infty[: g(x) \prec 0$

$g(x) \prec 0 \Leftrightarrow \ln(1+x) \prec x$.

$\forall x \in]0,+\infty[: 1+x \succ 1$: $\ln(1+x) \succ \ln 1 = 0$

$\ln$ [illegible] $]0,+\infty[$.

$\forall x \in]0,+\infty[: 0 \prec \ln(1+x) \prec x$. (0,5)

II- f [illegible] : $f(x) = x + \ln\left(\frac{x+1}{x-1}\right)$.

(1 D [illegible] f :

$$x \in D \Leftrightarrow \begin{cases} x-1 \neq 0 \\ \frac{x+1}{x-1} \succ 0 \end{cases} \Leftrightarrow (x+1)(x-1) \succ 0 \Leftrightarrow x \in]-\infty,-1[\cup]1,+\infty[$$

$D =]-\infty,-1[\cup]1,+\infty[$. (0,5)

(2 f [illegible] 0 :

$$\forall x \in D : f(-x) = -x + \ln\left(\frac{-x+1}{-x-1}\right) = -x + \ln\left(\frac{x-1}{x+1}\right)$$

$$\forall x \in D : \ln\left(\frac{x-1}{x+1}\right) = -\ln\left(\frac{x+1}{x-1}\right)$$

$\forall x \in D : f(-x) = -f(x)$.

f [illegible] . (0,5)

$-f$: $\lim_{x\to+\infty} \frac{x+1}{x-1} = 1 \Rightarrow \lim_{x\to+\infty} \ln\left(\frac{x+1}{x-1}\right) = \ln 1 = 0$

$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} x = +\infty$.

$\lim_{x\to1^+} \frac{x+1}{x-1} = \lim_{x\to1^+} \frac{2}{x-1} = +\infty$: $\lim_{x\to1^+} \ln\left(\frac{x+1}{x-1}\right) = +\infty$

$\lim_{x\to1^+} f(x) = 1 + \lim_{x\to1^+} \ln\left(\frac{x+1}{x-1}\right) = +\infty$. (0,5)

(3 $-$:

$$\forall x \in D : f'(x) = (x)' + \frac{\left(\frac{x+1}{x-1}\right)'}{\left(\frac{x+1}{x-1}\right)} = 1 + \frac{\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}}{(x-1)^2} \times \frac{x-1}{x+1}$$

$$f'(x) = 1 - \frac{2}{(x-1)(x+1)}$$

$\forall x \in D : f'(x) = \frac{x^2-1-2}{x^2-1} = \frac{x^2-3}{x^2-1}$. (0,75)

$-f$: $\forall x \in D : f'(x) = \frac{(x+\sqrt{3})(x-\sqrt{3})}{x^2-1}$

: $\forall x \in]1,+\infty[: sg[f'(x)] = sg[x-\sqrt{3}]$ [illegible] f

$[\sqrt{3},+\infty[$ [illegible] $]1,\sqrt{3}]$. (0,5)

(4 $-$: $\lim_{x\to\pm\infty} f(x) - x = \lim_{x\to\pm\infty} \ln\left(\frac{x+1}{x-1}\right) = 0$

$\lim_{x\to\pm\infty} \frac{x+1}{x-1} = 1 \Rightarrow \lim_{x\to\pm\infty} \ln\left(\frac{x+1}{x-1}\right) = \ln 1 = 0$.

(Δ) [illegible] $y = x$ [illegible] (C) . (0,25)

$-f$: $\forall x \in D : \frac{x+1}{x-1} - 1 = \frac{2}{x-1}$

$\forall x \in]-\infty,-1[: \frac{x+1}{x-1} \prec 1$, $\forall x \in]1,+\infty[: \frac{x+1}{x-1} \succ 1$:

$\forall x \in]-\infty,-1[: \ln\left(\frac{x+1}{x-1}\right) \prec \ln 1 = 0$, $\forall x \in]1,+\infty[: \ln\left(\frac{x+1}{x-1}\right) \succ \ln 1 = 0$

(0,5)

[illegible] 1 - :

$\forall x \in]1,+\infty[: f(x) \succ x$ [illegible] $\forall x \in]-\infty,-1[: f(x) \prec x$

$]1,+\infty[$ [illegible] (Δ) [illegible] (C) [illegible] $]-\infty,-1[$ [illegible]

(0,25) . (Δ) [illegible] (C) [illegible]

(6 [illegible] : $\begin{cases} u(x) = \ln\left(\dfrac{x+1}{x-1}\right) \\ v'(x) = 1 \end{cases}$ [illegible] : $\begin{cases} u'(x) = \dfrac{-2}{x^2-1} \\ v(x) = x \end{cases}$

[illegible] : $\displaystyle\int_2^4 \ln\left(\frac{x+1}{x-1}\right)dx = \left[x\ln\left(\frac{x+1}{x-1}\right)\right]_2^4 + \int_2^4 \frac{2x}{x^2-1}dx$

(1,25) $\displaystyle = \left[x\ln\left(\frac{x+1}{x-1}\right) + \ln(x^2-1)\right]_2^4$

[illegible] : $\displaystyle\int_2^4 \ln\left(\frac{x+1}{x-1}\right)dx = 4\ln\left(\frac{5}{3}\right) + \ln(15) - 3\ln(3) = 5\ln5 - 6\ln3$.

[illegible] (C) [illegible] $y = x$ [illegible] $x = 4$ [illegible] $x = 2$

(0,5) . [illegible] : $\displaystyle\int_2^4 [f(x) - x]dx = \int_2^4 \ln\left(\frac{x+1}{x-1}\right)dx = (5\ln5 - 6\ln3)cm^2$

-III [illegible] $(u_n)_{n\geq 2}$ [illegible] : $\forall n \geq 2 : u_n = f(n) - n$.

(1 [illegible] : $\forall n \geq 2 : u_n = \ln\left(\dfrac{n+1}{n-1}\right)$ [illegible] $\dfrac{n+1}{n-1} = \dfrac{n-1+2}{n-1} = 1 + \dfrac{2}{n-1}$

(0,25) . $\forall n \geq 2 : u_n = \ln\left(1 + \dfrac{2}{n-1}\right)$: [illegible]

[illegible] : $\forall n \geq 2 : \dfrac{2}{n-1} \succ \dfrac{2}{n}$ [illegible] : $\forall n \geq 2 : \dfrac{2}{n-1} \succ \dfrac{2}{n+1-1}$

[illegible] : $1 + \dfrac{2}{n-1} \succ 1 + \dfrac{2}{n+1-1}$ [illegible] $\ln\left(1+\dfrac{2}{n-1}\right) \succ \ln\left(1+\dfrac{2}{n+1-1}\right)$

(0,75) . [illegible] $(u_n)_{n\geq 2}$ [illegible] : $\forall n \geq 2 : u_n \succ u_{n+1}$ [illegible]

(2 [illegible] (I 2) [illegible] :

$\forall n \geq 2 : 0 \prec \ln\left(1 + \dfrac{2}{n-1}\right) \prec \dfrac{2}{n-1}$ ([illegible] $x = \dfrac{2}{n-1}$)

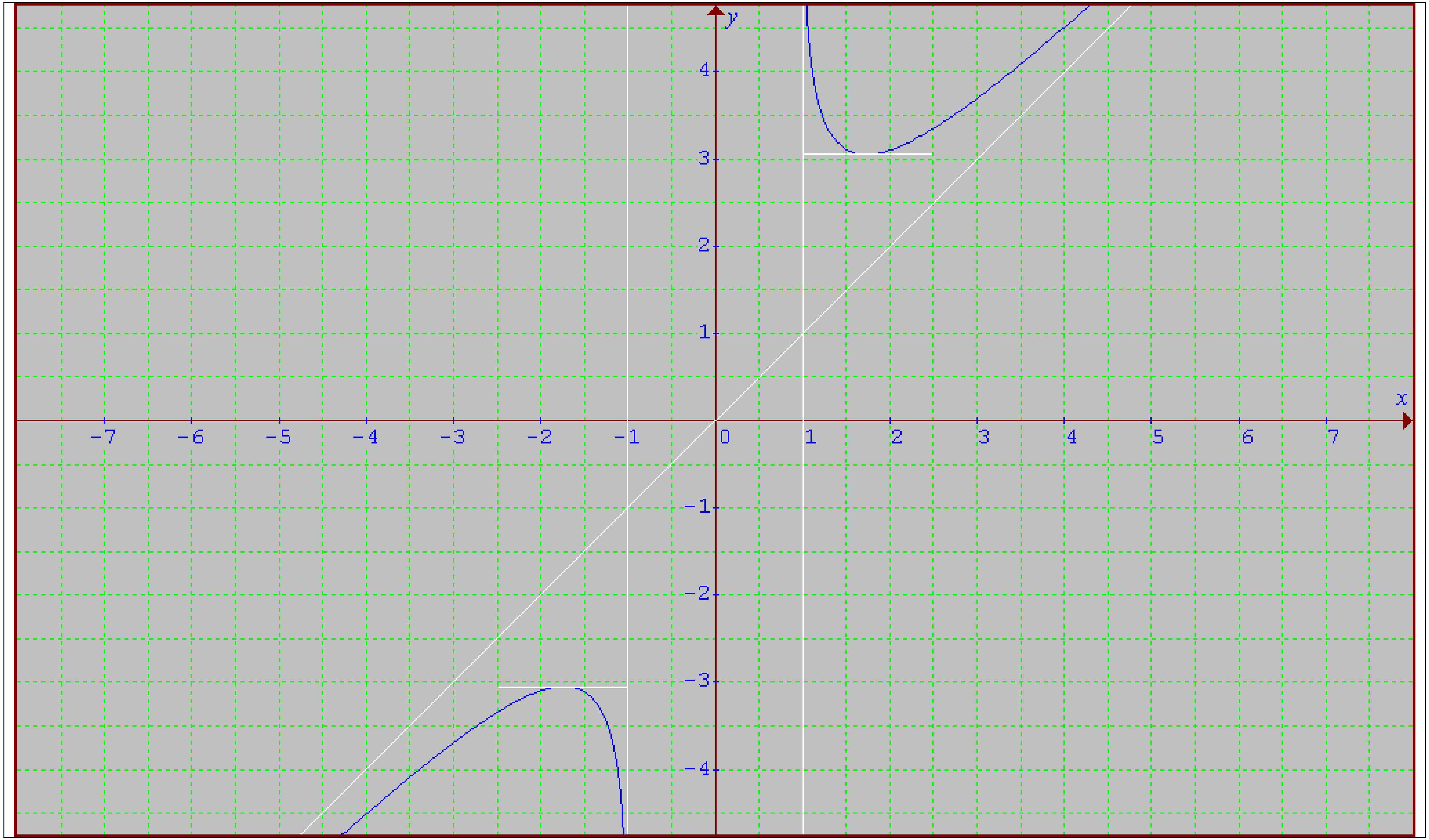
(0,5) . $\forall n \geq 2 : 0 \prec u_n \prec \dfrac{2}{n-1}$: [illegible]

[illegible] $(u_n)_{n\geq 2}$ [illegible] :

$\forall n \geq 2 : 0 \prec u_n \prec \dfrac{2}{n-1}$ [illegible] $\displaystyle\lim_{n\to+\infty} \frac{2}{n-1} = 0$: [illegible] $\displaystyle\lim_{n\to+\infty} u_n = 0$.

(0,5)

(5 [illegible] (C) : (1)



y
x
4
3
2
1
0
-1
-2
-3
-4
-7
-6
-5
-4
-3
-2
-1
1
2
3
4
5
6
7