ثانوية محمد السادس ورزازات	الحساب المثلثي *** ملخص ***	الأستاذ : الحيان المستوى: الثانية بكالوريا علوم

$\forall x\in IR-\left\{\frac{\pi}{2}+k\pi \;/\; K\in Z\right\};\tan(x)=\frac{\sin(x)}{\cos(x)}$
$\forall x\in IR-\left\{k\pi \;/\; K\in Z\right\};\cot an(x)=\frac{\cos(x)}{\sin(x)}$
$\forall x\in IR-\left\{\frac{\pi}{2}+k\pi \;/\; K\in Z\right\};\left\{\begin{array}{l}\cos^2(x)=\frac{1}{1+\tan^2(x)}\\ \sin^2(x)=\frac{\tan^2(x)}{1+\tan^2(x)}\end{array}\right.$
$\forall x\in IR;$ $-1\le\cos(x)\le 1$; $-1\le\sin(x)\le 1$; $\lvert\cos(x)\rvert\le 1$; $\lvert\sin(x)\rvert\le 1$
$\forall x\in IR;\cos^2(x)+\sin^2(x)=1$
$\forall x\in IR;\cos(-x)=\cos(x)$: cos دالة زوجية $\forall x\in IR;\sin(-x)=-\sin(x)$: sin دالة فردية $\forall x\in IR-\left\{\frac{\pi}{2}+k\pi \;/\; K\in Z\right\};\tan(-x)=-\tan(x)$: tan دالة فردية $\forall x\in IR-\left\{k\pi \;/\; k\in Z\right\};\cot an(-x)=-\cot an(x)$: $\cot an$ دالة فردية
$\forall x\in IR;$ $\cos(\pi-x)=-\cos(x)$; $\cos(\frac{\pi}{2}-x)=\sin(x)$; $\cos(\pi+x)=-\cos(x)$; $\cos(\frac{\pi}{2}+x)=-\sin(x)$ $\sin(\pi-x)=\sin(x)$; $\sin(\frac{\pi}{2}-x)=\cos(x)$; $\sin(\pi+x)=-\sin(x)$; $\sin(\frac{\pi}{2}+x)=\cos(x)$
$\forall x\in IR-\left\{\frac{\pi}{2}+k\pi \;/\; k\in Z\right\};$ $\tan(\pi-x)=-\tan(x)$; $\tan(x+\pi)=\tan(x)$; $\tan(x-\pi)=\tan(x)$
$\forall x\in IR-\left\{k\pi \;/\; k\in Z\right\};$ $\cot an(\pi-x)=-\cot an(x)$; $\cot an(x+\pi)=\cot an(x)$; $\cot an(x-\pi)=\cot an(x)$
$\forall x\in IR-\left\{\frac{\pi}{2}+k\pi ,k\pi \;/\; k\in Z\right\};$ $\tan(\frac{\pi}{2}-x)=\cot an(x)$; $\tan(\frac{\pi}{2}+x)=-\cot an(x)$; $\cot an(\frac{\pi}{2}-x)=\tan(x)$; $\cot an(\frac{\pi}{2}+x)=-\tan(x)$

$$\forall x \in IR - \left\{ \frac{\pi}{2} + k\pi \ / \ k \in Z \right\}, \forall h \in Z; \boxed{\tan(x + h\pi) = \tan(x)}$$

$$\forall x \in IR - \{ k\pi \ / \ k \in Z \}, \forall h \in Z; \boxed{\cot an(x + h\pi) = \cot an(x)}$$

$$\forall x \in IR - \{ k\pi \ / \ k \in Z \}; \boxed{1 + \cot an^2(x) = \frac{1}{\sin^2(x)}} \quad \forall x \in IR - \left\{ \frac{\pi}{2} + k\pi \ / \ k \in Z \right\}; \boxed{1 + \tan^2(x) = \frac{1}{\cos^2(x)}}$$

$$\forall x \in IR, \forall k \in Z; \quad \boxed{\cos(x + 2\pi) = \cos(x)} \quad \boxed{\cos(x + 2k\pi) = \cos(x)}$$

$$\boxed{\sin(x + 2\pi) = \sin(x)} \quad \boxed{\sin(x + 2k\pi) = \sin(x)}$$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
$\tan(x)$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\times$	0
$\cot an(x)$	$\times$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$\times$

$$\boxed{\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b)} \quad \boxed{\sin(a + b) = \sin(a)\cos(b) + \cos(a)\sin(b)}$$

$$\boxed{\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b)} \quad \boxed{\sin(a - b) = \sin(a)\cos(b) - \cos(a)\sin(b)}$$

$$\boxed{\cos(2a) = \cos^2(a) - \sin^2(a)}$$

$$\boxed{\cos(2a) = 2\cos^2(a) - 1} \qquad \boxed{\sin(2a) = 2\sin(a)\cos(a)}$$

$$\boxed{\cos(2a) = 1 - 2\sin^2(a)}$$

$$\boxed{\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a + b) + \cos(a - b)\right]}$$

$$\boxed{\sin(a)\sin(b) = -\frac{1}{2}\left[\cos(a + b) - \cos(a - b)\right]}$$

$$\boxed{\sin(a)\cos(b) = \frac{1}{2}\left[\sin(a + b) + \sin(a - b)\right]}$$

$$\boxed{\cos(a)\sin(b) = \frac{1}{2}\left[\sin(a + b) - \sin(a - b)\right]}$$

$$\boxed{\tan(a + b) = \frac{\tan(a) + \tan(b)}{1 - \tan(a)\tan(b)}} \qquad \boxed{\tan(2a) = \frac{2\tan(a)}{1 - \tan^2(a)}}$$

$$\boxed{\tan(a - b) = \frac{\tan(a) - \tan(b)}{1 + \tan(a)\tan(b)}}$$

$$\cos^2(a)=\frac{1+\cos(2a)}{2}$$

$$\sin^2(a)=\frac{1-\cos(2a)}{2}$$

$$\tan(a)=\frac{1-\cos(2a)}{1+\cos(2a)}$$

$$1+\cos(a)=2\cos^2\left(\frac{a}{2}\right)$$

$$1-\cos(a)=2\sin^2\left(\frac{a}{2}\right)$$

$$\sin(a)=2\sin\left(\frac{a}{2}\right)\cos\left(\frac{a}{2}\right)$$

$$\cos(a)+\cos(b)=2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\cos(a)-\cos(b)=-2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\sin(a)+\sin(b)=2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\sin(a)-\sin(b)=2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

اذا وضعنا $t=\tan\left(\frac{x}{2}\right)$, فاننا نحصل على :

$$\cos(x)=\frac{1-t^2}{1+t^2} \qquad \sin(x)=\frac{2t}{1+t^2} \qquad \tan(x)=\frac{2t}{1-t^2}$$

$$\tan(a)+\tan(b)=\frac{\sin(a+b)}{\cos(a)\cos(b)}$$

$$\tan(a)-\tan(b)=\frac{\sin(a-b)}{\cos(a)\cos(b)}$$

$$\cos(x)=\cos(\alpha)\Leftrightarrow\begin{cases}x=\alpha+2k\pi\\ ou \\ x=-\alpha+2k\pi\end{cases} \quad / k\in Z$$

$$\sin(x)=\sin(\alpha)\Leftrightarrow\begin{cases}x=\alpha+2k\pi\\ ou \\ x=\pi-\alpha+2k\pi\end{cases} \quad / k\in Z$$

$$\tan(x)=\tan(\alpha)\Leftrightarrow x=\alpha+k\pi \;/ k\in Z$$

$$\cot an(x)=\cot an(\alpha)\Leftrightarrow x=\alpha+k\pi \;/ k\in Z$$

$$a\cos(x)+b\sin(x)=\sqrt{a^2+b^2}\cos(x-\alpha)$$ حيث α عدد حقيقي يحقق :

$$\cos(\alpha)=\frac{a}{\sqrt{a^2+b^2}} \quad \text{و} \quad \sin(\alpha)=\frac{b}{\sqrt{a^2+b^2}}$$

$$a\cos(x)+b\sin(x)=\sqrt{a^2+b^2}\sin(x+\alpha)$$ حيث α عدد حقيقي يحقق :

$$\sin(\alpha)=\frac{a}{\sqrt{a^2+b^2}} \quad \text{و} \quad \cos(\alpha)=\frac{b}{\sqrt{a^2+b^2}}$$

اذا كانت M نقطة من الدائرة المثلثية U , وكان x أفصولا منحنيا لها, فان $(\cos(x),\sin(x))$ هو زوج احداثيتي النقطة M بالنسبة للمعلم $(O,\vec{OI},\vec{OJ})$ ولدينا :

$$\vec{OM}=\cos(x).\vec{OI}+\sin(x).\vec{OJ}$$

ليكن ABC مثلثا . نضع : $a=BC$ و $b=CA$ و $c=AB$ و $\hat{A}=\widehat{CAB}$ و $\hat{B}=\widehat{ABC}$ و $\hat{C}=\widehat{BCA}$

S : مساحة المثلث ABC . R : شعاع الدائرة المحيطة بالمثلث ABC . r : شعاع الدائرة المحاطة بالمثلث ABC .

p : نصف محيط الثلث ABC . أي : $p=\dfrac{a+b+c}{2}$.

لدينا :

$$\frac{\sin(\widehat{A})}{a}=\frac{\sin(\widehat{B})}{b}=\frac{\sin(\widehat{C})}{c}=\frac{2S}{abc}=\frac{1}{2R}=\frac{2pr}{abc}$$

$$\cos(\widehat{A})=\frac{b^2+c^2-a^2}{2bc} \qquad \cos(\widehat{B})=\frac{c^2+a^2-b^2}{2ca} \qquad \cos(\widehat{C})=\frac{b^2+a^2-c^2}{2ba}$$

$$S=\frac{1}{2}bc\cos(\widehat{A})=\frac{1}{2}ca\cos(\widehat{B})=\frac{1}{2}ab\cos(\widehat{C})=\frac{abc}{4R}=pr$$

<u>Formule d'Héron</u> : $S=\sqrt{p(p-a)(p-b)(p-c)}$ **: <u>صيغة هيرون</u>**

$$\tan(\widehat{A})=\frac{4S}{b^2+c^2-a^2} \qquad \tan(\widehat{B})=\frac{4S}{c^2+a^2-b^2} \qquad \tan(\widehat{C})=\frac{4S}{a^2+b^2-c^2}$$

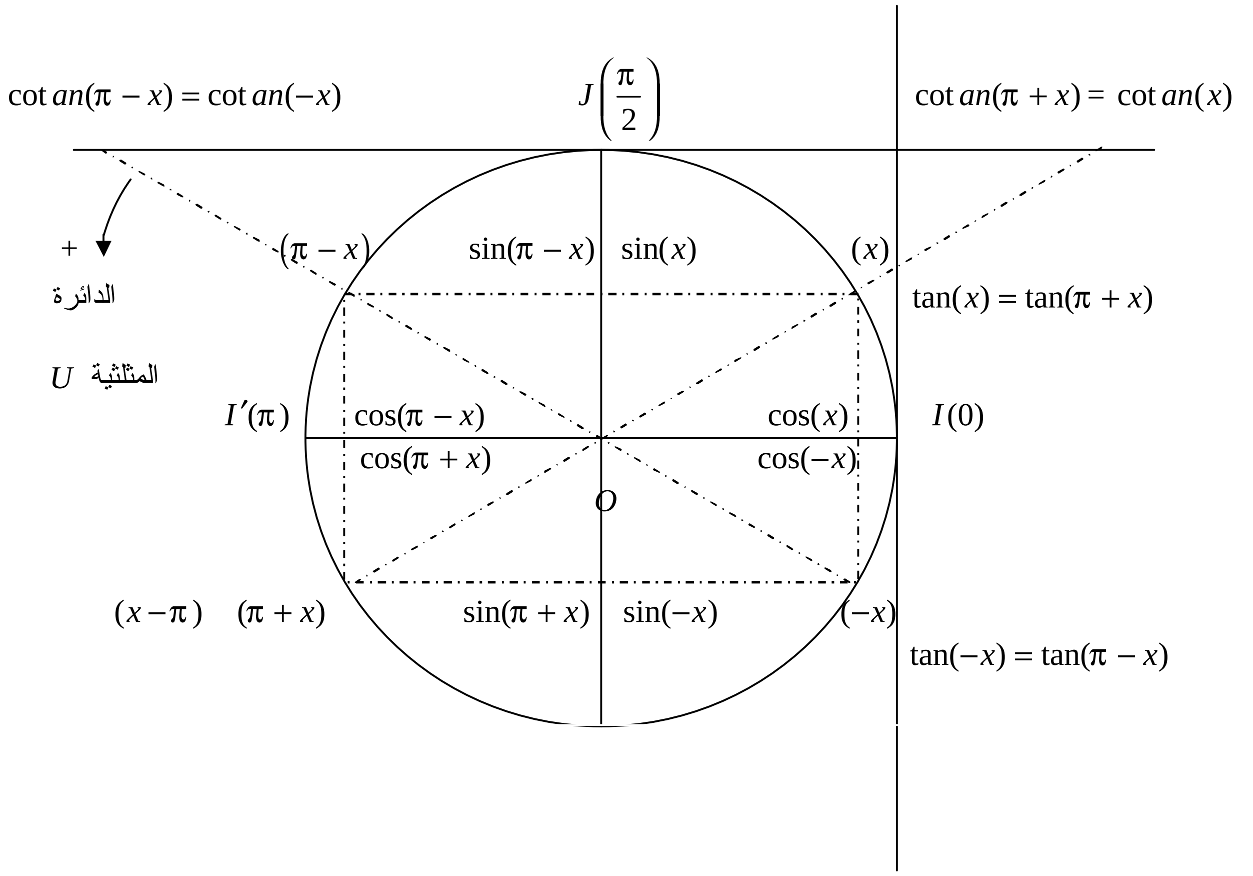
$$S=\frac{a^2}{2}\frac{\sin(\widehat{B}).\sin(\widehat{C})}{\sin(\widehat{B}+\widehat{C})}=\frac{b^2}{2}\frac{\sin(\widehat{C}).\sin(\widehat{A})}{\sin(\widehat{C}+\widehat{A})}=\frac{c^2}{2}\frac{\sin(\widehat{A}).\sin(\widehat{B})}{\sin(\widehat{A}+\widehat{B})}$$

واذا كان r_a و r_b و r_c , على التوالي, شعاع الدائرة J_a و J_b و J_c المحاطة خارجيا بالمثلث ABC ,

(r_a, r_b, r_c, sont les rayons respectifs des cercles exinscrits J_a ,, $J_b$et J_c au triangle ABC)

فان :

$$S=(p-a)r_a=(p-b)r_b=(p-c)r_c=\sqrt{r.r_a.r_b.r_c}$$


$\cot an(\pi - x) = \cot an(-x)$
$J\left(\frac{\pi}{2}\right)$
$\cot an(\pi + x) = \cot an(x)$
+
الدائرة
U المثلثية
$(\pi - x)$
$\sin(\pi - x)$
$\sin(x)$
(x)
$\tan(x) = \tan(\pi + x)$
$I'(\pi)$
$\cos(\pi - x)$
$\cos(x)$
$I(0)$
$\cos(\pi + x)$
$\cos(-x)$
O
$(x - \pi)$
$(\pi + x)$
$\sin(\pi + x)$
$\sin(-x)$
$(-x)$
$\tan(-x) = \tan(\pi - x)$