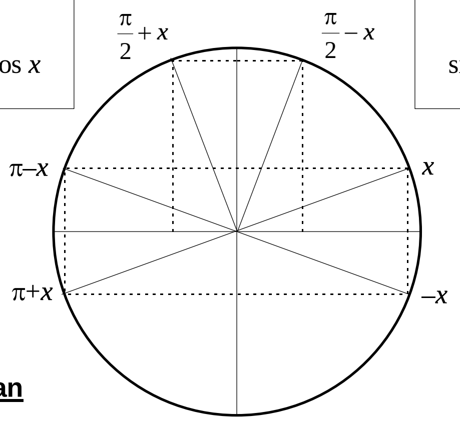
TRIGONOMÉTRIE : FORMULAIRE

Angles associés

Une lecture efficace du cercle trigonométrique permet de retrouver les relations suivantes :

$$\cos\left(\frac{\pi}{2}+x\right) = -\sin x \qquad \sin\left(\frac{\pi}{2}+x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2}-x\right) = \sin x \qquad \sin\left(\frac{\pi}{2}-x\right) = \cos x$$



$$\cos(\pi - x) = -\cos x \qquad \sin(\pi - x) = \sin x$$

$$\cos(\pi + x) = -\cos x \qquad \sin(\pi + x) = -\sin x$$

$$\cos(-x) = \cos x \qquad \sin(-x) = -\sin x$$

Relations entre cos, sin et tan

$$\cos^2 x + \sin^2 x = 1 \qquad 1 + \tan^2 x = \frac{1}{\cos^2 x}$$

Formules d'addition

$$\cos(a - b) = \cos a \cos b + \sin a \sin b \qquad \cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a - b) = \sin a \cos b - \cos a \sin b \qquad \sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b} \qquad \tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

Formules de duplication

$$\cos(2a) = \cos^2 a - \sin^2 a = 2\cos^2 a - 1 = 1 - 2\sin^2 a \qquad \sin(2a) = 2 \sin a \cos a \qquad \tan(2a) = \frac{2\tan a}{1 - \tan^2 a}$$

Extensions : $\cos(3a) = 4\cos^3 a - 3\cos a$ $\quad$ $\sin(3a) = 3\sin a - 4\sin^3 a$ $\quad$ $\tan(3a) = \frac{3\tan a - \tan^3 a}{1 - 3\tan^2 a}$

Au delà, utiliser la formule de Moivre.

Formules de linéarisation

$$\cos^2 a = \frac{1 + \cos(2a)}{2} \qquad \sin^2 a = \frac{1 - \cos(2a)}{2} \qquad \tan^2 a = \frac{1 - \cos(2a)}{1 + \cos(2a)}$$

Extensions : $\cos^3 a = \frac{\cos(3a) + 3\cos a}{4}$ $\quad$ $\sin^3 a = \frac{-\sin(3a) + 3\sin a}{4}$ $\quad$ $\tan^3 a = \frac{-\sin(3a) + 3\sin a}{\cos(3a) + 3\cos a}$

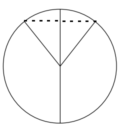
Au delà, utiliser les formules d'Euler. Les formules d'Euler permettent également de montrer que :

$$\cos a \cos b = \frac{1}{2}[\cos(a - b) + \cos(a + b)] \qquad \cos a \sin b = \frac{1}{2}[\sin(a + b) - \sin(a - b)] \qquad \sin a \sin b = \frac{1}{2}[\cos(a - b) - \cos(a + b)]$$

$$\cos p + \cos q = 2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right) \qquad \cos p - \cos q = -2\sin\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$$

$$\sin p + \sin q = 2\sin\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right) \qquad \sin p - \sin q = 2\sin\left(\frac{p-q}{2}\right)\cos\left(\frac{p+q}{2}\right)$$

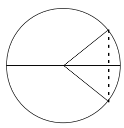
Résolution d'équations trigonométriques



$$\cos U = \cos V \iff (U = V\ [2\pi] \text{ ou } U = -V\ [2\pi])$$

$$\sin U = \sin V \iff (U = V\ [2\pi] \text{ ou } U = \pi - V\ [2\pi])$$

$$\tan U = \tan V \iff U = V\ [\pi]$$



Expression du cosinus, sinus et tangente en fonction de la tangente de l'angle moitié

Si $t = \tan\frac{a}{2}$, on a : $\quad \cos a = \frac{1 - t^2}{1 + t^2}$; $\sin a = \frac{2t}{1 + t^2}$; $\tan a = \frac{2t}{1 - t^2}$